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MATHEMATICAL SIMULATION OF METAL SOLIDIFICATION IN A WEDGE-LIKE CASTING MOLD WITH ALLOWANCE FOR NATURAL CONVECTION

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Using the finite difference method, a nonstationary problem of metal solidification in a wedge-like casting mold has been solved in a two-dimensional statement with allowance for natural convection. For isolated instants of time the positions of the solidification front, the profiles of temperature, horizontal and vertical velocity components, and of the stream function have been obtained.

Keywords: *convection velocity, solidification, temperature profiles, casting mold, liquid phase, solid phase, heat-transfer equation, phase transition, characteristic dimensions, velocity curl.*

Introduction. Allowance for convection in nonlinear problems of solidification greatly complicates the process of obtaining a solution. Therefore, historically the influence of convection was considered in steps, with a statement of problems and solution methods made gradually more complex. Among the first works where convection was taken into account mention may be made of work [1], in which the equations of heat conduction, Navier–Stokes for one velocity component, and continuity equations were considered in a dimensionless form. With the aid of power series with undetermined coefficients for the temperature and velocities in a liquid phase, an approximate distribution of temperatures, heat fluxes, and of velocities was obtained for a semi-infinite slab. Numerical calculations were carried out for two points under the assumption of linear motion of the solidification front. In [2], convection in a semi-infinite slab was investigated on the basis of studying hydrodynamic equations alone. The proposed approximate solutions for the velocities of a descending and ascending flows made it possible to construct the profiles of vertical velocities and present a hydrodynamic picture in the form of two vortices with vertical velocity maxima at the solidification front and in the ingot center. The values of velocities were found within the range 0.1–0.4 m/sec. In [3], free convection in the liquid phase of a continuous ingot was studied at a constant rate of pouring into a crystallizer. The problem was solved numerically, and the temperature profiles, stream function isolines, and velocity profiles were obtained. The influence of the two-phase zone was taken into account, and the values of velocities of up to 2 cm/sec were obtained. The influence of forced convection on the character of solidification was investigated in [4]. Heat conduction equations for two phases were considered; the velocities were written provided the boundary conditions and the continuity equation were complied with. In using the notion of the local thermodynamic potential and the variational formulation of the problem with subsequent term-by-term integration, an analytical approximate solution for the front motion velocity was obtained. The numerical calculations based on the equation obtained made it possible to explain the appearance of various crystalline zones inside a plane ingot. Turbulent convection in a solidifying ingot was investigated in [5]. A system of vortices adjoining the solidification front and attenuation of vortices in the thermally insulated upper portion of the ingot were revealed. The possibility of controlling convection was studied in [6]. It was shown that for a setup with directed crystallization the onset of eddy motion at the crystallization front was impossible if the Grashof number exceeded 10^6. No determination of velocity profiles was made in [5, 6].

In the present work we consider the influence of temperature profiles on the vertical and horizontal velocities of convection and the motion of the ingot solidification front in a plane wedge-like casting mold. The problem is

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solved numerically by the finite difference method using a computer program developed by the present authors. The casting mold represents a truncated wedge and has the angle of inclination φ of the side faces to the vertical plane. The cross section of the wedge constitutes a trapezoid, with the side faces representing rectangles. It is assumed that the wedge length is much greater than its thickness, and the influence of the end face surfaces of the wedge is not taken into account.

Statement of the Problem. We write the equation of heat transfer for an incompressible fluid [7]:

$$\frac{\partial T}{\partial t} + \nabla (\overline{v}T) = a\nabla^2 T . \tag{1}$$

The Navier–Stokes equations in dimensional quantities for the vertical and horizontal velocity components [8] are

$$\frac{\partial \overline{u}}{\partial \overline{t}} + \overline{u}\frac{\partial \overline{u}}{\partial \overline{x}} + \overline{v}\frac{\partial \overline{u}}{\partial \overline{y}} = -\frac{1}{\overline{\rho}}\frac{\partial \overline{P}}{\partial \overline{x}} + \frac{\overline{\eta}_v}{\overline{\rho}}\left(\frac{\partial^2 \overline{u}}{\partial \overline{x}^2} + \frac{\partial^2 \overline{u}}{\partial \overline{y}^2}\right) - \beta (T - T_m)\,\overline{g}\sin\varphi , \tag{2}$$

$$\frac{\partial \overline{v}}{\partial \overline{t}} + \overline{u}\frac{\partial \overline{v}}{\partial \overline{x}} + \overline{v}\frac{\partial \overline{v}}{\partial \overline{y}} = -\frac{1}{\overline{\rho}}\frac{\partial \overline{P}}{\partial \overline{y}} + \frac{\overline{\eta}_v}{\overline{\rho}}\left(\frac{\partial^2 \overline{v}}{\partial \overline{x}^2} + \frac{\partial^2 \overline{v}}{\partial \overline{y}^2}\right) + \beta (T - T_m)\,\overline{g}\cos\varphi . \tag{3}$$

A curvilinear orthogonal coordinate system [7, 9] and continuity equation for two velocity components are used:

$$\frac{\partial \overline{u}}{\partial \overline{x}} + \frac{\partial \overline{v}}{\partial \overline{y}} = 0 . \tag{4}$$

The following conditions are considered: for temperature $T = T_b = T_w$ on the side wall and on the bottom of the mold, $\frac{\partial T}{\partial x} = 0$ on the vertical axis of the mold, and $T = T_m$ at the crystallization front; for velocities $u = v = 0$ at the crystallization front and $\frac{\partial v}{\partial x} = 0$ on the vertical axis of the mold.

Solution of the Problem. According to the method of alternating directions, the heat conduction equation in finite differences has the form [7]
at $i\uparrow$, $j\uparrow$

$$T_{i,j}^{n+1} = T_{i,j}^{n} - \frac{\tau}{2\xi} u_{i,j}^{n} (T_{i+1,j}^{n} - T_{i-1,j}^{n}) - \frac{\tau}{2\eta} v_{i,j}^{n} (T_{i,j+1}^{n} - T_{i,j-1}^{n})$$

$$+ \frac{\tau}{\xi^2} a (T_{i+1,j}^{n} - T_{i,j}^{n} - T_{i,j}^{n+1} + T_{i-1,j}^{n+1}) + \frac{\tau}{\eta^2} a (T_{i,j+1}^{n} - T_{i,j}^{n} - T_{i,j}^{n+1} + T_{i,j-1}^{n+1}) , \tag{5}$$

whence

$$T_{i,j}^{n+1} = \frac{1}{1 + a\tau\left(\frac{1}{\xi^2} + \frac{1}{\eta^2}\right)}\left[T_{i,j}^{n} - \frac{\tau}{2\xi} u_{i,j}^{n} (T_{i+1,j}^{n} - T_{i-1,j}^{n}) - \frac{\tau}{2\eta} v_{i,j}^{n} (T_{i,j+1}^{n} - T_{i,j-1}^{n})\right.$$

$$\left. + \frac{\tau}{\xi^2} a (T_{i+1,j}^{n} - T_{i,j}^{n} + T_{i-1,j}^{n+1}) + \frac{\tau}{\eta^2} a (T_{i,j+1}^{n} - T_{i,j}^{n} + T_{i,j-1}^{n+1})\right] ; \tag{6}$$

at $i\downarrow$, $j\downarrow$

$$T_{i,j}^{n+1} = \frac{1}{1 + a\tau\left(\frac{1}{\xi^2} + \frac{1}{\eta^2}\right)}\left[T_{i,j}^{n} - \frac{\tau}{2\xi}u_{i,j}^{n}(T_{i+1,j}^{n} - T_{i-1,j}^{n}) - \frac{\tau}{2\eta}v_{i,j}^{n}(T_{i,j+1}^{n} - T_{i,j-1}^{n})\right.$$

$$\left. + \frac{\tau}{\xi^2}a(T_{i+1,j}^{n+1} - T_{i,j}^{n} + T_{i-1,j}^{n}) + \frac{\tau}{\eta^2}a(T_{i,j+1}^{n+1} - T_{i,j}^{n} + T_{i,j-1}^{n})\right]; \tag{7}$$

at $i\uparrow$, $j\downarrow$

$$T_{i,j}^{n+1} = \frac{1}{1 + a\tau\left(\frac{1}{\xi^2} + \frac{1}{\eta^2}\right)}\left[T_{i,j}^{n} - \frac{\tau}{2\xi}u_{i,j}^{n}(T_{i+1,j}^{n} - T_{i-1,j}^{n}) - \frac{\tau}{2\eta}v_{i,j}^{n}(T_{i,j+1}^{n} - T_{i,j-1}^{n})\right.$$

$$\left. + \frac{\tau}{\xi^2}a(T_{i+1,j}^{n} - T_{i,j}^{n} + T_{i-1,j}^{n+1}) + \frac{\tau}{\eta^2}a(T_{i,j+1}^{n+1} - T_{i,j}^{n} + T_{i,j-1}^{n+1})\right]; \tag{8}$$

at $i\downarrow$, $j\uparrow$

$$T_{i,j}^{n+1} = \frac{1}{1 + a\tau\left(\frac{1}{\xi^2} + \frac{1}{\eta^2}\right)}\left[T_{i,j}^{n} - \frac{\tau}{2\xi}u_{i,j}^{n}(T_{i+1,j}^{n} - T_{i-1,j}^{n}) - \frac{\tau}{2\eta}v_{i,j}^{n}(T_{i,j+1}^{n} - T_{i,j-1}^{n})\right.$$

$$\left. + \frac{\tau}{\xi^2}a(T_{i+1,j}^{n+1} - T_{i,j}^{n} + T_{i-1,j}^{n}) + \frac{\tau}{\eta^2}a(T_{i,j+1}^{n} - T_{i,j}^{n} + T_{i,j-1}^{n+1})\right]. \tag{9}$$

In solving the heat conduction equation the phase transition is taken into account as follows. The specific heat removed from a cell at the initial moment is $q_{i,j}^{0} = 0$. If $T^{n+1} \le T_{\mathrm{m}}$ and $T^{n} > T_{\mathrm{m}}$, as well as $q \le \lambda$, the specific heat removed from the cell at the time step τ is

$$q_{\mathrm{p}} = c\,(T^{n} - T^{n+1}),$$

the specific heat evolved in the cell in the process of crystallization at the time step τ is

$$q_{\mathrm{m}} = \lambda - q .$$

Next, if $q_{\mathrm{p}} > q_{\mathrm{m}}$, then

$$T^{n+1} = T^{n+1} - \frac{q_{\mathrm{p}} - q_{\mathrm{m}}}{c}, \quad q = \lambda ,$$

otherwise

$$T^{n+1} = T_{\mathrm{m}}, \quad q = q + q_{\mathrm{p}} - c\,(T - T_{\mathrm{m}}) .$$

Differentiating Eq. (2) with respect to y and Eq. (3) with respect to x, excluding the pressure [8] and defining the curl as

$$\bar{\zeta} = \frac{\partial \bar{u}}{\partial \bar{y}} - \frac{\partial \bar{v}}{\partial \bar{x}}, \tag{10}$$

we obtain the equation of curl transform:

$$\frac{\partial \overline{\zeta}}{\partial \overline{t}} = -\overline{u}\,\frac{\partial \overline{\zeta}}{\partial \overline{x}} - \overline{v}\,\frac{\partial \overline{\zeta}}{\partial \overline{y}} + \frac{\overline{\eta}_v}{\overline{\rho}}\left(\frac{\partial^2 \overline{\zeta}}{\partial \overline{x}^2} + \frac{\partial^2 \overline{\zeta}}{\partial \overline{y}^2}\right) - \beta \overline{g}\left(\frac{\partial T}{\partial \overline{x}}\cos\varphi + \frac{\partial T}{\partial \overline{y}}\sin\varphi\right). \tag{11}$$

This equation in a conservative form is [8]

$$\frac{\partial \overline{\zeta}}{\partial \overline{t}} = -\frac{\partial\,(\overline{u}\overline{\zeta})}{\partial \overline{x}} - \frac{\partial\,(\overline{v}\overline{\zeta})}{\partial \overline{y}} + \frac{\overline{\eta}_v}{\overline{\rho}}\left(\frac{\partial^2 \overline{\zeta}}{\partial \overline{x}^2} + \frac{\partial^2 \overline{\zeta}}{\partial \overline{y}^2}\right) - \beta \overline{g}\left(\frac{\partial T}{\partial \overline{x}}\cos\varphi + \frac{\partial T}{\partial \overline{y}}\sin\varphi\right). \tag{12}$$

Defining the stream function $\overline{\psi}$ by the relations

$$\frac{\partial \overline{\psi}}{\partial \overline{y}} = \overline{u}\,, \quad \frac{\partial \overline{\psi}}{\partial \overline{x}} = -\,\overline{v}\,, \tag{13}$$

we write Eq. (10) in the form

$$\frac{\partial^2 \overline{\psi}}{\partial \overline{x}^2} + \frac{\partial^2 \overline{\psi}}{\partial \overline{y}^2} = \overline{\zeta}\,. \tag{14}$$

The dimensional quantities are barred from above. We introduce the dimensions and nondimensionalize the equations. Then the equation of curl transform (12) and Eq. (10) in a dimensionless form can be represented as

$$\frac{\partial \zeta}{\partial t} = -\frac{\partial\,(u\zeta)}{\partial x} - \frac{\partial\,(v\zeta)}{\partial y} + \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 \zeta}{\partial x^2} + \frac{\partial^2 \zeta}{\partial y^2}\right) - \beta g\left(\frac{\partial T}{\partial x}\cos\varphi + \frac{\partial T}{\partial y}\sin\varphi\right), \tag{15}$$

$$\zeta = \frac{\partial u}{\partial y} - \frac{\partial u}{\partial x}\,. \tag{16}$$

In finite differences Eq. (15) is
at $i\uparrow$, $j\uparrow$

$$\zeta_{i,j}^{n+1} = \zeta_{i,j}^{n} - \frac{\tau}{2\xi}\,(u_{i+1,j}^{n}\zeta_{i+1,j}^{n} - u_{i-1,j}^{n}\zeta_{i-1,j}^{n}) - \frac{\tau}{2\eta}\,(v_{i,j+1}^{n}\zeta_{i,j+1}^{n} - v_{i,j-1}^{n}\zeta_{i,j-1}^{n})$$

$$+\frac{\tau}{\mathrm{Re}}\left(\frac{\zeta_{i+1,j}^{n} - \zeta_{i,j}^{n} - \zeta_{i,j}^{n+1} + \zeta_{i-1,j}^{n+1}}{\xi^2} + \frac{\zeta_{i,j+1}^{n} - \zeta_{i,j}^{n} - \zeta_{i,j}^{n+1} + \zeta_{i,j-1}^{n+1}}{\eta^2}\right)$$

$$-\,\beta\tau g\left(\frac{T_{i+1,j}^{n} - T_{i-1,j}^{n}}{2\xi}\cos\varphi + \frac{T_{i,j+1}^{n} - T_{i,j-1}^{n}}{2\eta}\sin\varphi\right),$$

whence

$$\zeta_{i,j}^{n+1} = \frac{1}{1 + \dfrac{\tau}{\mathrm{Re}}\left(\dfrac{1}{\xi^2} + \dfrac{1}{\eta^2}\right)}\Bigg[\zeta_{i,j}^{n} - \frac{\tau}{2\xi}\,(u_{i+1,j}^{n}\zeta_{i+1,j}^{n} - u_{i-1,j}^{n}\zeta_{i-1,j}^{n})$$

$$-\,\frac{\tau}{2\eta}\,(v_{i,j+1}^{n}\zeta_{i,j+1}^{n} - v_{i,j-1}^{n}\zeta_{i,j-1}^{n}) + \frac{\tau}{\mathrm{Re}}\left(\frac{\zeta_{i+1,j}^{n} - \zeta_{i,j}^{n} + \zeta_{i-1,j}^{n+1}}{\xi^2} + \frac{\zeta_{i,j+1}^{n} - \zeta_{i,j}^{n} + \zeta_{i,j-1}^{n+1}}{\eta^2}\right)$$

$$- \beta\tau g\left(\frac{T^n_{i+1,j} - T^n_{i-1,j}}{2\xi}\cos\varphi + \frac{T^n_{i,j+1} - T^n_{i,j-1}}{2\eta}\sin\varphi\right)\Bigg];$$

at $i\downarrow,\ j\downarrow$

$$\zeta^{n+1}_{i,j} = \frac{1}{1 + \frac{\tau}{\mathrm{Re}}\left(\frac{1}{\xi^2} + \frac{1}{\eta^2}\right)}\Bigg[\zeta^n_{i,j} - \frac{\tau}{2\xi}\,(u^n_{i+1,j}\zeta^n_{i+1,j} - u^n_{i-1,j}\zeta^n_{i-1,j})$$

$$- \frac{\tau}{2\eta}\,(v^n_{i,j+1}\zeta^n_{i,j+1} - v^n_{i,j-1}\zeta^n_{i,j-1}) + \frac{\tau}{\mathrm{Re}}\left(\frac{\zeta^{n+1}_{i+1,j} - \zeta^n_{i,j} + \zeta^n_{i-1,j}}{\xi^2} + \frac{\zeta^{n+1}_{i,j+1} - \zeta^n_{i,j} + \zeta^n_{i,j-1}}{\eta^2}\right)$$

$$- \beta\tau g\left(\frac{T^n_{i+1,j} - T^n_{i-1,j}}{2\xi}\cos\varphi + \frac{T^n_{i,j+1} - T^n_{i,j-1}}{2\eta}\sin\varphi\right)\Bigg];$$

at $i\uparrow,\ j\downarrow$

$$\zeta^{n+1}_{i,j} = \frac{1}{1 + \frac{\tau}{\mathrm{Re}}\left(\frac{1}{\xi^2} + \frac{1}{\eta^2}\right)}\Bigg[\zeta^n_{i,j} - \frac{\tau}{2\xi}\,(u^n_{i+1,j}\zeta^n_{i+1,j} - u^n_{i-1,j}\zeta^n_{i-1,j})$$

$$- \frac{\tau}{2\eta}\,(v^n_{i,j+1}\zeta^n_{i,j+1} - v^n_{i,j-1}\zeta^n_{i,j-1}) + \frac{\tau}{\mathrm{Re}}\left(\frac{\zeta^n_{i+1,j} - \zeta^n_{i,j} + \zeta^{n+1}_{i-1,j}}{\xi^2} + \frac{\zeta^{n+1}_{i,j+1} - \zeta^n_{i,j} + \zeta^n_{i,j-1}}{\eta^2}\right)$$

$$- \beta\tau g\left(\frac{T^n_{i+1,j} - T^n_{i-1,j}}{2\xi}\cos\varphi + \frac{T^n_{i,j+1} - T^n_{i,j-1}}{2\eta}\sin\varphi\right)\Bigg];$$

at $i\downarrow,\ j\uparrow$

$$\zeta^{n+1}_{i,j} = \frac{1}{1 + \frac{\tau}{\mathrm{Re}}\left(\frac{1}{\xi^2} + \frac{1}{\eta^2}\right)}\Bigg[\zeta^n_{i,j} - \frac{\tau}{2\xi}\,(u^n_{i+1,j}\zeta^n_{i+1,j} - u^n_{i-1,j}\zeta^n_{i-1,j})$$

$$- \frac{\tau}{2\eta}\,(v^n_{i,j+1}\zeta^n_{i,j+1} - v^n_{i,j-1}\zeta^n_{i,j-1}) + \frac{\tau}{\mathrm{Re}}\left(\frac{\zeta^{n+1}_{i+1,j} - \zeta^n_{i,j} + \zeta^n_{i-1,j}}{\xi^2} + \frac{\zeta^n_{i,j+1} - \zeta^n_{i,j} + \zeta^{n+1}_{i,j-1}}{\eta^2}\right)$$

$$- \beta\tau g\left(\frac{T^n_{i+1,j} - T^n_{i-1,j}}{2\xi}\cos\varphi + \frac{T^n_{i,j+1} - T^n_{i,j-1}}{2\eta}\sin\varphi\right)\Bigg].$$

The stream function ψ is found by the Richardson iteration method [8]:

$$\psi^{k+1}_{i,j} = \frac{1}{2\,(1 + \beta^2)}\,(\psi^k_{i+1,j} + \psi^k_{i-1,j} + \beta^2\psi^k_{i,j+1} + \beta^2\psi^k_{i,j-1} - \xi^2\zeta_{i,j})\,.$$

The initial conditions are taken in the form

$$u^0_{i,j} = 0\,;\quad v^0_{i,j} = 0\,;\quad \zeta^0_{i,j} = 0\,;\quad \psi^0_{i,j} = 0\,.$$

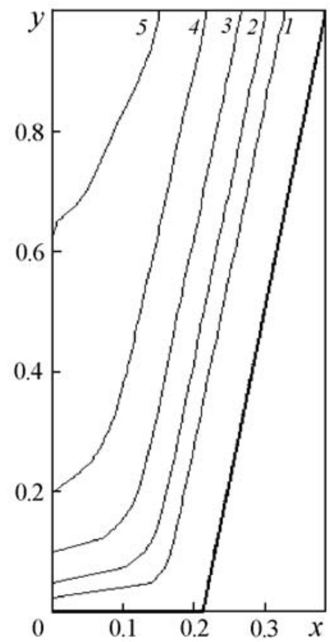

Fig. 1. The positions of the solidification front in a wedge-like casting mold for time instants: 1) 200 sec; 2) 400; 3) 800; 4) 1600; 5) 3200 at a superheating temperature of 10 K. The right side of the vertical section is shown. Straight solid lines depict the inner side wall and bottom of the mold, as well as the thermally insulated upper surface of metal.

The boundary conditions [8] for the wall $i = l$ with sticking are

$$\psi_{l,j} = 0\,; \quad \zeta_{l,j} = 2\,\frac{\psi_{l,j+1} - \psi_{l,j}}{\eta^2}\,,$$

and for the symmetry plane (a wall with slipping) are

$$\psi_{0,j} = 0\,; \quad \zeta_{0,j} = \zeta_{1,j}\,.$$

At the angular point at the apex of the concave angle of the step these conditions are

$$\psi_{l,j} = 0\,; \quad \zeta_{l,j} = 0\,.$$

At the angular point at the apex of the convex angle of the step they are

$$\psi_{l,j} = 0\,; \quad \zeta_{l,j} = 2\,\frac{\psi_{l,j+1}}{\eta^2} + 2\,\frac{\psi_{l+1,j}}{\xi^2}\,.$$

Numerical calculations were carried out for the following parameters of the mold and liquid steel: R_1 = 1.2 m; $\overline{L}$ = 1 m; $\varphi = 10^{\circ}$; T_m = 1808 K; $T_b = T_w$ = 1768 K; T_1 = 1818 K; $\overline{\rho}$ = 7310 kg/m^3; $\overline{g}$ = 9.81 m/sec^2; $\lambda = 2.72 \cdot 10^5$ J/kg; $a_1 = 5.7 \cdot 10^{-6}$ m^2/sec; $a_2 = 4.84 \cdot 10^{-6}$ m^2/sec; $\beta = 0.06 \cdot 10^{-3}$ 1/K; c_1 = 800 J/(kg·K); c_2 = 660 J/(kg·K); $\overline{\eta}_v = 4.4 \cdot 10^{-3}$ kg/(m·sec); U_0 = 0.96 mm/sec.

The right half of the mold was divided into cells with 40 steps along the radius and 160 along the azimuth; the time step was τ = 0.75 sec, the step of iterations was τ_1 = 0.09 msec. The relative error of iterations was 10^{-6}. The relative error caused by the temperature dependence of the liquid phase density was equal to $2.5 \cdot 10^{-3}$. Figure 1

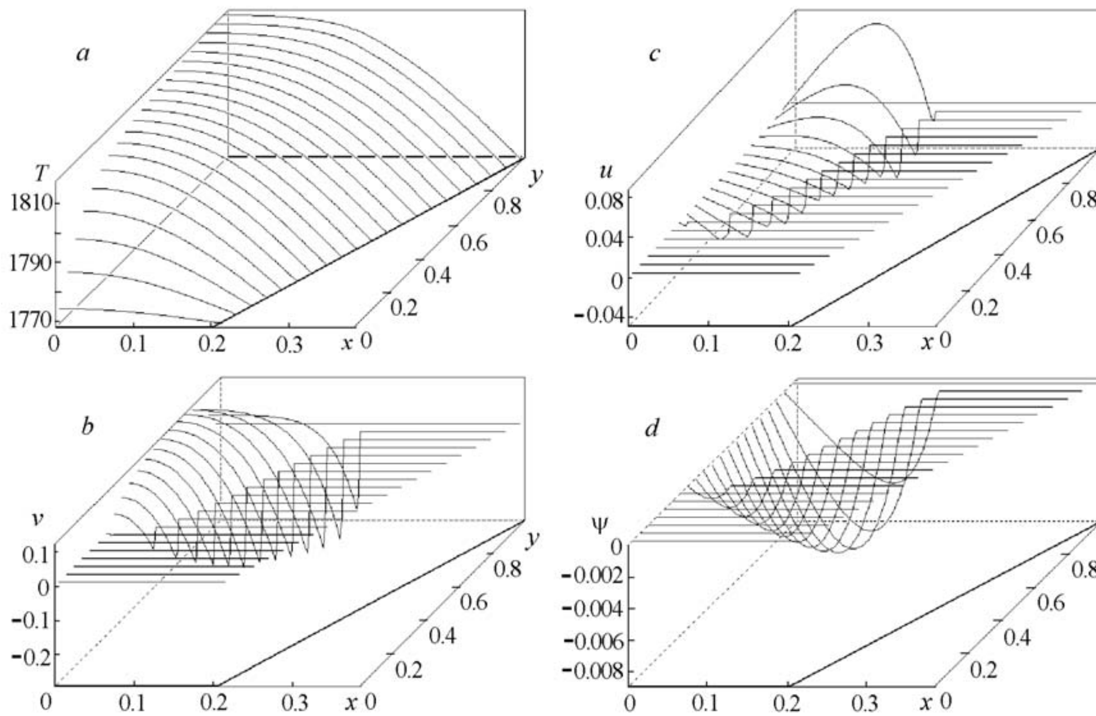

Fig. 2. The profiles of temperature calculated for the time instant t = 2000 sec in the horizontal sections of the mold in the liquid and solid phases (a), of the vertical velocity component (b), horizontal velocity component (c), and of the values of the stream function (d) in the liquid phase.

shows the positions of the solidification front for a number of instants of time. The profiles of temperature (Fig. 2a), vertical velocity (Fig. 2b), horizontal velocity (Fig. 2c), and of the stream function (Fig. 2d) were calculated.

Conclusions. Analyzing the advancement of the solidification front (Fig. 1), we may conclude that its velocity gradually decreases. As is seen from Fig. 2a, the maximum temperature gradients are localized on the solidification front. Due to this, the vertical velocity profiles (Fig. 2b) have sharply expressed maxima near the solidification front and smooth maxima in the central zone of the ingot. The horizontal velocity components (Fig. 2c) are much smaller in magnitude and are directed from the solidification front near its boundary, whereas in the center of the ingot — to the solidification front. From this it follows that a closed single vortex with a maximum velocity of 0.3 mm/sec is formed in the liquid region. Approximately the same convection velocities were obtained in [10]. This also follows from the graph of the stream function (Fig. 2d). In these calculations, the Reynolds number $\mathrm{Re} = \frac{\overline{U}_0\overline{L}\overline{\rho}}{\overline{\eta}_v} = 1600$, the Grashof number $\mathrm{Gr} = \frac{\overline{\rho}^2\overline{g}\overline{L}^3}{\overline{\eta}_v^2}\beta\Delta T = 16\cdot10^9$, the Prandtl number $\mathrm{Pr} = \frac{\overline{\eta}_v}{\overline{a}_1\overline{\rho}} = 0.105$, and the Nusselt number $\mathrm{Nu} = 0.53\sqrt[4]{\mathrm{Gr}\cdot\mathrm{Pr}^2} = 61.5$ [11].

NOTATION

a, thermal diffusivity; a_1, a_2, thermal diffusivity of the liquid and solid phases, $\mathrm{m^2/sec}$; c, specific heat; c_1, c_2, specific heats of the liquid and solid phases, J/(kg·K); Gr, Grashof number; $\overline{g}$, g, dimensionless and dimensional free fall acceleration; $\overline{L}$, height of the casting mold, m; Nu, Nusselt number; $\overline{P}$, pressure, Pa; Pr, Prandtl number; q, specific heat removed from the cell at the given instant of time, J/kg; q_m, specific heat evolved in the cell during crystallization in time step, J/kg; q_p, specific heat removed from the cell in time step, J/kg; R_1, radius of the mold bottom, m; Re, Reynolds number; $\overline{T}$, temperature, K; T_1, initial temperature of metal, K; T_m, crystallization temperature, K; T_b, T_w, temperature of the bottom and side wall of the mold, K; $\bar{t}$, t, dimensional and dimensionless time, sec; $\bar{U}_0$, characteristic velocity, m/sec; $\bar{u}$, u, dimensional and dimensionless horizontal velocity component, m/sec; $\bar{v}$, v, dimensional and dimensionless vertical velocity component, m/sec; $\bar{x}$, $\bar{y}$, x, y, dimensional and dimensionless coordinates, m; β, coefficient of volumetric thermal expansion, K^{-1}; $\bar{\zeta}$, ζ, dimensional and dimensionless values of a vortex, sec^{-1}; η, step of iterations over the vertical; $\bar{\eta}_v$, dynamic viscosity of a melt, kg/(m·sec); λ, specific heat of melting, J/kg; ξ, step of iterations along the horizontal; ρ, density of a metal, kg/m^3; τ, step of iterations in time; φ, conicity angle of the mold, deg; $\bar{\psi}$, ψ, dimensional and dimensionless stream function, m^2/sec. Subscripts and superscripts: b, bottom of the mold; i, j, numbers of cells along the horizontal and vertical; l, number along the horizontal of the cell corresponding to the solidification front; k, number of iteration; m, melting; n, number of step in time; p, specific heat; v, viscosity; w, side wall.

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